論　文

An Economic Analysis of “Semi-public” Loss Prevention

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Abstract

The main purpose of this study is to investigate the semi-public loss prevention, wherein only some individuals undertake public loss prevention, while all individuals undertake private loss prevention. This study builds an economic model for examining the optimal private loss prevention for all individuals and the optimal fee amount to be paid to the association that collects a fee from some individuals and undertakes public loss prevention.

Keywords: Loss prevention, Economic analysis

1. Introduction

In the field of risk management, loss prevention is one of the important aspects requiring investigation. The main problem is that individuals might not voluntarily undertake loss prevention.

In the field of microeconomic theory, individuals would undertake loss prevention only when their expected utility from doing so is larger than that obtained by not doing so.

There are many studies that investigate the loss prevention. It is often discussed in economic models in insurance because an insurance contract influences not only the risk allocation, but also the incentive schemes, such as deductibles and experience rating, to achieve loss prevention. The pioneering studies in this research field are, Holmstrom (1979), Raviv (1979), Shavell (1979), and Rubinstein and Yaari (1983).[1]

These literatures focus mainly on the effect of an individual’s loss prevention. They focus on situations in which an individual incurs a loss prevention cost and himself or herself enjoys the benefit from loss prevention. Thus, it is called “private” loss prevention.

In contrast, Lee (1992) built a model that includes the loss prevention undertaken by the government. In this model, the government collects the tax from all individuals and undertakes loss prevention. In contrast to private loss prevention, the moral hazard problem is alleviated because the government compels individuals who do not want to voluntarily undertake loss prevention to pay tax. However, each individual cannot undertake his or her optimal loss prevention if there are differences in wealth, utility, and loss prevention cost functions; this is because the loss prevention undertaken by the government is the same across all individuals. This study focuses on the situation in which all individuals incur a loss prevention cost and enjoy the benefit from loss prevention. Thus, in Lee (1992), it is called "public" loss prevention.

Although there are studies that have investigated private and public loss prevention, there is another kind of loss prevention—called "semi-public" loss prevention—in the real world. "Semi-public" implies that only some individuals undertake public loss prevention, while all individuals undertake private loss prevention.

The most plausible example of semi-public loss prevention is the loss prevention undertaken by the Japan Traffic Safety Association (JTSA). According to the webpage of JTSA, the main function of JTSA is to engage in activities promoting traffic safety.[2] For example, JTSA gives yellow caps and hats to small children; because such children appear prominent and can be easily spotted by drivers, the probability of road accidents occurring is lowered. To conduct such activities, JTSA collects a fee from the individuals who drive automobiles. Although they are not obliged to pay this fee, some of them pay this fee when they renew their driver's license. From the viewpoint of loss prevention, the individuals who drive automobiles indirectly undertake public loss prevention by paying the fee to JTSA. In contrast, individuals who cannot drive automobiles—for instance, small children—never commit public loss prevention because they do not pay the fee. Further, we know that all individuals undertake private loss prevention for their own benefits.

The purpose of this study is to investigate such semi-public loss prevention through a microeconomic model. This study is organized as follows. Section 2 develops a model containing the individuals who can, and cannot, drive automobiles and the association that collects the fee from individuals who can. The equilibrium of this model is derived in section 3. Comparative statics are presented in section 4 and the main results of this study are derived. Section 5 has concluding remarks.

2. The Model

Suppose that there are two types of individuals. The first type includes those who have an opportunity to drive automobiles (hereafter, "type A" individuals). The second type includes those who never drive automobiles (hereafter, "type N" individuals). Assume that the total number of individuals is normalized to one and $n \in (0,1)$ represents the ratio of type A individuals to the total population. Assume that each individual is either a driver or pedestrian. Further, assume that all accidents occur because of a collision between automobile (driver) and pedestrian. $\gamma \in (0,1)$ indicates the ratio of hours spent driving to total hours of type A individuals. In contrast, type N individuals are always pedestrians because they never drive automobiles. Furthermore, there is an association that collects a fee from type A individuals and undertakes public loss prevention for pedestrians through activities, such as giving yellow caps and hats to children. The following two-stage game is considered.

In the first stage, the association decides the fee amount. For the sake of simplicity, all type A individuals pay the same fee amount, denoted by $f>0$. Thus, the total fee amount can be written as nf. However, the association also uses the fee for other purposes, such as paying salaries of employees in the association. $\theta \in (0,1)$ denotes the proportion of its budget that the association spends on loss prevention. The association invests the fee it collects on activities that lower pedestrian's private loss prevention costs. Such loss prevention benefits all pedestrians, that is, both types of individuals. Then, the loss prevention conducted by the association can be understood as public loss prevention.

In the second stage, after observing the fee amount, both types of individuals decide their level of private loss prevention. Let e_j^i be the level of private loss prevention. $i \in \{A,N\}$ represents an individual's type and $j \in \{D,P\}$ represents an individual's status, where D and P denote "driver(s)" and "pedestrian(s)", respectively.[3] k_D^A represents the private loss prevention cost function of drivers and the form of this function is assumed to be

$$k_D^A = \frac{a_D e_D^{A^2}}{2}$$

where a_D>0. In contrast, k_P^i represents the private loss prevention cost function of pedestrians and its form is assumed to be

$$k_P^i = \frac{1}{2}\frac{a_P}{\sqrt{\theta nf}} e_P^{i^2} \ (1)$$

where a_P>0. θnf in equation (1) represents the association's investment, which is equal to the amount it spends on public loss prevention. Then, the association can lower the marginal cost of private loss prevention of pedestrians, but this effect is marginally decreasing.

Accident probability is denoted by $\pi \in [0,1]$. The accident probability depends on the ratio of type A and type N individuals; the ratio of hours spent driving to total hours of type A individuals, as represented by γ; and the level of private loss prevention for both types of individuals. In this model, accident probability is assumed to be

$$\pi = 2n\gamma(1-n\gamma)\left[1 - e_D^A - \left\{\frac{n(1-\gamma)}{n(1-\gamma)+(1-n)} e_P^A + \frac{1-n}{n(1-\gamma)+(1-n)} e_P^N\right\}\right]. \ (2)$$

The explanation for equation (2) is as follows. First, $2n\gamma(1-n\gamma)$ represents the probability of an automobile colliding with a pedestrian. Second, the brackets in equation (2) indicate the effect of private loss prevention. The more the private loss prevention, the lower the accident probability. The braces in equation (2) indicate the expected level of private loss prevention of pedestrians.

If an accident occurs, both driver and pedestrian suffer damage. x_D and x_P represent the level of damage when the individual is a driver and pedestrian, respectively.

From the above setting, the expected costs for each type of individual, which are denoted by c^i, can be written as follows.

$$c^A = \gamma(\pi x_D + k_D^A) + (1-\gamma)(\pi x_P + k_P^A) + f, (3)$$

$$c^N = \pi x_P + k_P^N. \ (4)$$

Further, we assume that all individuals want to minimize the expected costs shown in equations (3) and (4). In contrast, the association wants to minimize the following total costs that are denoted by c.

$$c = nc^A + (1-n)c^N. \ (5)$$

3. Deriving the Equilibrium

In the second stage, the first order conditions can be written as

$$\frac{\partial c^A}{\partial e_D^A} = \gamma[a_D e_D^{A*} - 2n(1-n\gamma)\{\gamma x_D + (1-\gamma)x_P\}] = 0, (6)$$

$$\frac{\partial c^A}{\partial e_P^A} = \frac{(1-\gamma)\left[a_P e_P^{A*} - 2n^2\gamma\sqrt{\theta nf}\{\gamma x_D + (1-\gamma)x_P\}\right]}{\sqrt{\theta nf}} = 0, (7)$$

$$\frac{\partial c^N}{\partial e_P^N} = \frac{a_P e_P^{N*}}{\sqrt{\theta nf}} - 2n(1-n)\gamma x_P = 0 \ (8)$$

where the asterisk represents the equilibrium value.[4] From equations (6) to (8), each optimal private loss prevention can be derived as

$$e_D^{A*} = \frac{2n(1-n\gamma)\{\gamma x_D + (1-\gamma)x_P\}}{a_D}, (9)$$

$$e_P^{A*} = \frac{2n^2\gamma\sqrt{\theta nf}\{\gamma x_D + (1-\gamma)x_P\}}{a_P}, (10)$$

$$e_P^{N*} = \frac{2n(1-n)\gamma\sqrt{\theta nf}x_P}{a_P}. (11)$$

For ensuring $\pi \in [0,1]$, we assume that both a_D and a_P are not small.

Substituting equations (9) through (11) into equations (3) and (4), we show

$$c^A = \frac{2n\gamma\{\gamma x_D + (1-\gamma)x_P\}}{a_D a_P}\left[-n(1-n\gamma)^2 a_P\{\gamma x_D + (1-\gamma)x_P\} + a_D\{(1-n\gamma)a_P - n\gamma\sqrt{\theta nf}A\}\right] + f, (12)$$

$$c^N = \frac{2n\gamma x_P}{\sqrt{\theta nf}a_D a_P}\left[-2n(1-n\gamma)^2\sqrt{\theta nf}a_P\{\gamma x_D + (1-\gamma)x_P\} + a_D\{(1-n\gamma)\sqrt{\theta nf}a_P - n^2\gamma\theta fB\}\right] \ (13)$$

where

$$A \equiv n^2\gamma(1-\gamma)x_D + [2 - n\{4 - n(3 - 2\gamma + \gamma^2)\}]x_P,$$

$$B \equiv 2n^2\gamma(1-\gamma)x_D + [1 - n\{2 - n(3 - 4\gamma + 2\gamma^2)\}]x_P.$$

In the first stage, the association chooses the optimal fee amount. By using equations (5), (12), and (13), the following first order condition can be derived.[5]

$$\frac{\partial c}{\partial f} = \frac{n\sqrt{\theta n f^*}a_P - n^3\gamma^2\theta[n^3\gamma^2(1-\gamma)x_D^2 + \{2n\gamma x_D + (1-n\gamma)x_P\}x_P C]}{\sqrt{\theta n f^*}a_P} = 0 \ (14)$$

where[6]

$$C \equiv 1 - n\{1 + \gamma - n(1 - \gamma + \gamma^2)\} > 0. (15)$$

From equation (14), we get

$$f^* = \frac{n^3\gamma^4\theta}{a_P^2}\Omega^2 \ (16)$$

where

$$\Omega \equiv n^3\gamma^2(1-\gamma)x_D^2 + \{2n\gamma x_D + (1-n\gamma)x_P\}x_P C > 0.$$

Substituting equation (16) in equations (10) and (11), the optimal private loss prevention when individuals are pedestrians can be derived as

$$e_P^{A*} = \frac{2n^4\gamma^3\theta\Omega\{\gamma x_D + (1-\gamma)x_P\}}{a_P^2}, (17)$$

$$e_P^{N*} = \frac{2n^3(1-n)\gamma^3\theta\Omega x_P}{a_P^2}. (18)$$

4. Comparative Statics

In this section, we demonstrate the effect of exogenous variables on the optimal fee amount and the private loss prevention represented in equations (9), (16), (17), and (18). This model has seven exogenous variables—a_D, a_P, x_D, x_P, θ, n, and γ. We can easily verify the effect of the former five exogenous variables and derive the following proposition.

Proposition 1: From comparative statics, the following results are derived. First, an increase in the marginal cost of driver's private loss prevention does not change the optimal fee amount and a pedestrian's optimal private loss prevention, while it decreases driver's optimal private loss prevention. Second, an increase in the marginal cost of a pedestrian's private loss prevention decreases the optimal fee amount and pedestrian's optimal private loss prevention, while it does not change driver's optimal private loss prevention. Third, an increase in the extent of damages increases the optimal fee amount, as well as the pedestrian's and driver's optimal private loss prevention. Fourth, an increase in the proportion of the budget that the association spends on public loss prevention increases the optimal fee amount and pedestrian's optimal private loss prevention, while it does not change driver's optimal private loss prevention.

The results described in proposition 1 are very intuitive. First, from equation (16), we find the marginal cost of driver's private loss prevention is not related to the optimal fee amount. Thus, only the drivers lead to decrease their optimal private loss prevention when the marginal cost of driver's private loss prevention increases. Second, an increase in the marginal cost of pedestrian's private loss prevention means that his or her private loss prevention becomes less efficient. Thus, the association decreases the optimal fee amount and pedestrian's optimal private loss prevention is lowered. Third, an increase in the extent of damages enhances the demand for private loss prevention. Then, the association increases the optimal fee amount and both pedestrians and drivers lead to increase their optimal private loss prevention. Fourth, an increase in the proportion of the budget that the association spends on public loss prevention means that the marginal effect of fee increases. Thus, the association leads to increase the optimal fee and then the pedestrians also increase optimal private loss prevention. However, the driver's optimal private loss prevention is not changed because a change in the fee amount is not related to the driver's private loss prevention.

In contrast, the effects of the remaining two exogenous variables, n and γ seem to be complicated. The following two propositions describe the effect of change in each exogenous variable on the optimal fee amount.

Proposition 2: In general, whether an increase in the ratio of type A individuals to the total population leads to an increase in the optimal fee amount is indeterminate. Then, γ <0.4929 is a sufficient condition for realizing that an increase in the ratio of type A individuals to the total population leads to an increase in the optimal fee amount.

Proof : see Appendix B

Proposition 3: In general, whether an increase in the ratio of hours spent driving to total hours of type A individuals increases the optimal fee amount is indeterminate. Then, $n<1/2$ and $\gamma<4/5$ are sufficient conditions for realizing that an increase in the ratio of hours spent driving to total hours of type A individuals leads to an increase in the optimal fee amount.

Proof : see Appendix C

The implications of both propositions are explained as follows. First, unlike the other five exogenous variables, we cannot derive a determinant result about the optimal fee amount because n and $1-n$ (γ and $1-\gamma$) coexist in equation (16). Second, sufficient conditions for realizing $\partial f^*/\partial n>0$ and $\partial f^*/\partial \gamma>0$ can be derived regardless of the extent of damages x_D and x_P. From the viewpoint of these sufficient conditions, the magnitudes of γ and n are key variables to decide the effect of these two exogenous variables.

By using the above two propositions, we can consider the effect of change in each exogenous variable on the optimal private loss prevention. From equations (10) and (11), we can derive the following equations if the conditions written in Propositions 2 and 3 are satisfied, that is, $\partial f^*/\partial n>0$ and $\partial f^*/\partial \gamma>0$ are realized.

$$\frac{\partial e_P^{A*}}{\partial n}>0,$$

$$\frac{\partial e_P^{N*}}{\partial n}>0 \quad if\ n>\frac{3}{5}, (19)$$

$$\frac{\partial e_P^{A*}}{\partial \gamma}>0 \quad if\ x_D \geq x_P, (20)$$

$$\frac{\partial e_P^{N*}}{\partial \gamma}>0.$$

In equations (19) and (20), $n>3/5$ and $x_D \geq x_P$ are sufficient conditions for satisfying these relations. In contrast, by differentiating equation (9) with respect to n and γ, we find that the signs of $\partial e_D^{A*}/\partial n$ and $\partial e_D^{A*}/\partial \gamma$ are indeterminate.

From the above analysis, we can derive the following proposition.

Proposition 4: First, increases in the ratio of type A individuals to the total population and the ratio of hours spent driving to total hours of type A individuals lead to an increase in the pedestrian's optimal private loss prevention when the conditions that are described in propositions 2 and 3 and additional sufficient conditions, such as $n>3/5$ and $x_D \geq x_P$, are satisfied. Second, whether increases in the ratio of type A individuals to the total population and the ratio of hours spent driving to total hours of type A individuals lead to an increase in the driver's optimal private loss prevention are indeterminate.

In light of Propositions 2 through 4, we find how the changes in the ratio of type A individuals to the total population and the ratio of hours spent driving to total hours of type A individuals affect the optimal fee amount and the optimal private loss are generally indeterminate. Under some conditions, we know that the increases in the ratio of type A individuals to the total population and the ratio of hours spent driving to total hours of type A individuals lead to an increase in the optimal fee amount and the pedestrian's optimal private loss prevention.

In the end, the results in the comparative statics can be summarized in the following table.

	a_D	a_P	x_D	x_P	θ	n	γ
e_D^{A*}	−	0	+	+	0	?	?
e_P^{A*}	0	−	+	+	+	+	+ ($x_D \geq x_P$)
e_P^{N*}	0	−	+	+	+	+ ($n>3/5$)	+
f^*	0	−	+	+	+	+	+

Notes:

1. "0" represents the case in which the equilibrium value is not changed when the exogenous variable changes.
2. "?" represents the effect is indeterminate.
3. All signs in column n indicate case when $\gamma<0.4929$.
4. All signs in column γ indicate case when $n<1/2$ and $\gamma<4/5$.

Table: The results in comparative statics

5. Concluding Remarks

This study focused on the semi-public loss prevention, wherein only some individuals undertake public loss prevention, while all individuals undertake private loss prevention. The main results of this study are summarized by the four propositions derived from comparative statics. Then, they find how to change the optimal fee amount and the private loss prevention when the exogenous situation is changed.

This study builds a simple model and we propose some possible extension of this model. For example, this model assumed that the individuals of each type are identical. Thus, for example, the ratio of hours spent driving to total hours of all type A individuals are same. If this assumption is relaxed, some results derived from this study might be changed.

Appendix A

In order to confirm $C>0$, it is sufficient to prove that the minimum value of C is strictly positive. In order to derive the minimum value of C, the first order conditions of equation (15), with respect to n and γ, are derived as follows.

$$\frac{\partial C}{\partial n} = -1 + \gamma + 2n\{1 - \gamma(1-\gamma)\} = 0, \text{(A1)}$$

$$\frac{\partial C}{\partial \gamma} = -n\{1 + n(1-2\gamma)\} = 0. \text{(A2)}$$

From equations (A1) and (A2), two kinds of solutions, that is, $\{n, \gamma\}=\{0,-1\}$ and $\{n, \gamma\}=\{1,1\}$ are derived. Second order conditions are

$$\frac{\partial^2 C}{\partial n^2} = 2\{1 - \gamma(1-\gamma)\},$$

$$\frac{\partial^2 C}{\partial \gamma^2} = 2n^2,$$

$$\frac{\partial^2 C}{\partial n^2}\frac{\partial^2 C}{\partial \gamma^2} - \left(\frac{\partial^2 C}{\partial n \partial \gamma}\right)^2 = -1 - 4n[1 - \{2 + 3n\gamma(1-\gamma)\}].$$

Then, we know that $\{n, \gamma\}=\{1,1\}$ is a unique solution to realize the minimum value of C because $\partial^2 C/\partial n^2=2>0$, $\partial^2 C/\partial \gamma^2=2>0$, and $(\partial^2 C/\partial n^2)(\partial^2 C/\partial \gamma^2)-(\partial^2 C/\partial n \partial \gamma)^2=3>0$. When $\{n, \gamma\}=\{1,1\}$, $C=0$ is realized. Thus, we can prove that $C>0$ is always satisfied because $n, \gamma<1$.

Q. E. D.

Appendix B

Differentiating equation (16) with respect to n, we show

$$\frac{\partial f^*}{\partial n} = \frac{3n^2\gamma^4\theta\Omega^2}{a_P^2} + \frac{2n^3\gamma^4\theta\Omega}{a_P^2}\frac{\partial \Omega}{\partial n} = \frac{n^2\gamma^4\theta\Omega}{a_P^2}\left(3\Omega + 2n\frac{\partial \Omega}{\partial n}\right). \text{(B1)}$$

From equation (B1), the sign of $\partial f^*/\partial n$ is always the same as that of $g \equiv 3\Omega+2n(\partial\Omega/\partial n)$.

The function g can be presented as

$$g = 9n^3\gamma^2(1-\gamma)x_D^2$$
$$+2n\gamma\{5 - 7n(1+\gamma) + 9n^2(1-\gamma+\gamma^2)\}x_D x_P$$
$$+[3 + n\{-5(1+2\gamma) + n(7 + 14\gamma^2 - 9n\gamma(1-\gamma+\gamma^2))\}]x_P^2. \text{(B2)}$$

The first term on the right-hand side of equation (B2) is always strictly positive, but the signs of second and third terms are indeterminate. Thus, in general, we cannot find the sign of g. In order to know the characteristics of g in detail, we check the condition under which $g>0$ tendency is to be realized.

From equation (B2), regardless of the magnitudes of x_D and x_P, the following two equations are sufficient conditions for realizing $g>0$.

$$g_2 \equiv 5 - 7n(1+\gamma) + 9n^2(1-\gamma+\gamma^2) \geq 0, \text{(B3)}$$

$$g_3 \equiv 3 + n\{-5(1+2\gamma) + n(7 + 14\gamma^2 - 9n\gamma(1-\gamma+\gamma^2))\} \geq 0. \text{(B4)}$$

From equation (B3), we find that g_2 is the convex quadratic function of n and one minimum value exists because $1-\gamma+\gamma^2>0$. Thus, it is sufficient to check the condition for realizing $g_2 \geq 0$ when g_2 has the minimum value. First order condition of g_2 with respect to n is

$$\frac{\partial g_2}{\partial n} = -7(1+\gamma) + 18n(1-\gamma+\gamma^2) = 0. \text{(B5)}$$

From equation (B5), we find the following value of n to minimize g_2.

$$n = \frac{7(1+\gamma)}{18(1-\gamma+\gamma^2)}. \text{(B6)}$$

In relation to equation (B6), the following lemma is indicated.

Lemma B1: Equation (B6) always exists in the range (0,1).

Proof:

$n>0$ is always satisfied because both numerator and denominator in equation (B6) are strictly positive. The condition in which $n<1$ is equivalent to $7(1+\gamma)<18(1-\gamma+$

γ^2). This inequality can be rewritten as $\gamma(25-18\gamma)<11$. The left-hand side of this inequality is a concave quadratic function of γ and one maximum value exists. $\gamma=25/36$ is the value to maximize the left-hand side of the inequality, then $625/72<11$ is confirmed.

Q. E. D.

Substituting equation (B6) in equation (B3), we get

$$g_2 = \frac{131}{36} - \frac{49\gamma}{12(1-\gamma+\gamma^2)}.$$

Solving $g_2=0$, the following solution is obtained.[7]

$$\gamma = \frac{1}{131}\left(139 - 12\sqrt{15}\right). \quad (B7)$$

Further, g_2 is a monotone decreasing function of γ in the range (0,1) because

$$\frac{\partial g_2}{\partial \gamma} = -\frac{49(1-\gamma^2)}{12(1-\gamma+\gamma^2)^2} < 0. \quad (B8)$$

From the results in equations (B7) and (B8), the sufficient condition for realizing $g_2 \geq 0$ shows

$$\gamma \leq \frac{1}{131}\left(139 - 12\sqrt{15}\right).$$

Next, we investigate g_3. Differentiating equation (B4) with respect to n, we get

$$\frac{\partial g_3}{\partial n} = -5(1+2\gamma) + n\{14(1+2\gamma^2) - 27n\gamma(1-\gamma+\gamma^2)\}. \quad (B9)$$

Second order condition can be computed as

$$\frac{\partial^2 g_3}{\partial n^2} = 14(1+2\gamma) - 54n\gamma(1-\gamma+\gamma^2). \quad (B10)$$

From equation (B9), we find that the following value for satisfying $\partial g_3/\partial n=0$.

$$n = \frac{7(1+2\gamma^2) - \sqrt{\Delta}}{27\gamma(1-\gamma+\gamma^2)} \quad (B11)$$

where

$$\Delta \equiv 49 + \gamma\{-135 + \gamma(61 + 135\gamma - 74\gamma^2)\}. \quad (B12)$$

We find that equation (B11) minimizes g_3 because $\partial^2 g_3/\partial n^2 = 2\sqrt{\Delta} > 0$). Before explaining the characteristics of g_3, the following lemma is indicated.

Lemma B2: Equation (B11) always exists in the range (0,1).

Proof:

In order to completely prove the lemma B2, three kinds of proofs must be combined.

The first proof is that the value of equation (B11) is not imaginary. This proof is equivalent to proving $\Delta \geq 0$. From equation (B12), Δ is a fourth-order function of γ. Differentiating Δ with respect to γ, we get

$$\frac{\partial \Delta}{\partial \gamma} = -296\gamma^3 + 405\gamma^2 + 122\gamma - 135. \quad (B13)$$

Further, second order condition is

$$\frac{\partial^2 \Delta}{\partial \gamma^2} = -888\gamma^2 + 810\gamma + 122. \quad (B14)$$

There are three solutions to satisfy $\partial\Delta/\partial\gamma=0$. By using equation (B13), we find that $\gamma \approx -0.5977$, $\gamma \approx 0.5321$, and $\gamma \approx 1.4339$ by Mathematica. Then, $\gamma \approx 0.5321$ is the only solution in the range (0,1). Substituting $\gamma=0.5321$ in equation (B14), we find $301.581>0$. This means that $\gamma \approx 0.5321$ realizes the minimum value of Δ in the range (0,1). Thus, it is sufficient to confirm that Δ is strictly positive when $\gamma \approx 0.5321$. Substituting $\gamma=0.5321$ in equation (B12), we find $\Delta=8.8437>0$. Thus, we know that the value of equation (B11) is not imaginary.

The second proof is that equation (B11) is always satisfied by $n>0$. It is sufficient to confirm that the numerator in equation (B11) is strictly positive because the denominator in equation (B11) is always strictly positive. We already know that $\gamma \approx 0.5321$ is the unique value that realizes the minimum value of Δ in the range (0,1). Thus, either $\gamma=0$ or $\gamma=1$ realizes the maximum value of Δ in the range (0,1). Substituting $\gamma=0$ and $\gamma=1$ in equation (B12) gives $\Delta=49$ and $\Delta=36$, respectively. Thus, we find $\gamma=0$ realizes the maximum value of Δ. In contrast, $7(1+2\gamma^2)$ in equation (B11) is minimized when $\gamma=0$. Thus, the numerator in equation (B11) is minimized when $\gamma=0$. Substituting $\gamma=0$ in the numerator of equation (B11) makes it zero. Thus, $n>0$ is always satisfied because $\gamma>0$.

The third proof is that $n<1$ is always satisfied. From equation (B11), the condition under which $n<1$ is realized can be written as follows.

$$27\gamma(1-\gamma+\gamma^2) - \left\{7(1+2\gamma^2) - \sqrt{\Delta}\right\} > 0. \quad (B15)$$

Rearranging equation (B15), we get

$$\sqrt{\Delta} > -27\gamma^3 + 41\gamma^2 - 27\gamma + 7. \quad (B16)$$

If the right-hand side of equation (B16) is non-positive, equation (B16) is surely satisfied because $\sqrt{\Delta}>0$.[8] Thus, only the case when the right-hand side of equation (B16) is strictly positive is confirmed. Because both sides in equation (B16) are strictly positive, the following inequality must be satisfied.

$$\Delta > (-27\gamma^3 + 41\gamma^2 - 27\gamma + 7)^2. \text{ (B17)}$$

Thus, equation (B17) is equivalent to

$$\Gamma \equiv \Delta - (-27\gamma^3 + 41\gamma^2 - 27\gamma + 7)^2 > 0. \text{ (B18)}$$

Rearranging the left-hand side of equation (B18), we get

$$\Gamma = 27\gamma(1-\gamma)(1-\gamma+\gamma^2)(9-28\gamma+27\gamma^2). \text{ (B19)}$$

In order to show $\Gamma>0$, we have to confirm $\Lambda \equiv 9-28\gamma+27\gamma^2>0$. Λ is a convex quadratic function of γ and it has one minimum value. Thus, it is sufficient to prove that the minimum value of Λ is strictly positive. We easily find that $\gamma=14/27$ realizes the minimum value of Λ and, then, $\Lambda=47/27>0$. *Q. E. D.*

By substituting equation (B11) in equation (B4), g_3 can be depicted by Mathematica, as shown in Figure B1.[9]

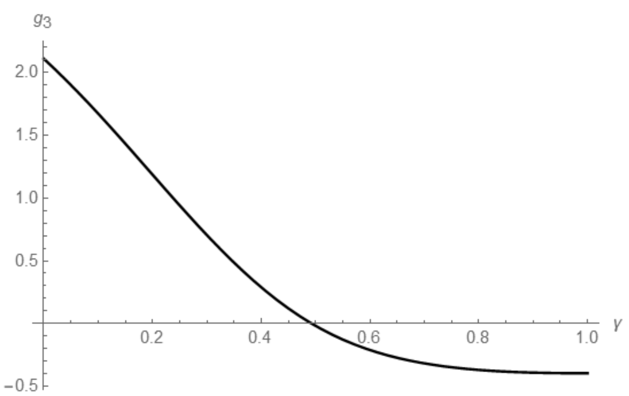

Figure B1: The function form of g_3

From Figure B1, we find that g_3 is positive (negative) when γ is relatively small (large). Then, we find that $g_3=0$ is realized when $\gamma \approx 0.4929$.

Finally, we find that equations (B3) and (B4) are always strictly positive when $\gamma<0.4929$ because $0.4929<(139-12\sqrt{15})/131$. Thus, we find that $\gamma<0.4929$ is a sufficient condition in which $\partial f^*/\partial n>0$ is always realized.

Appendix C

As in the case of n in Appendix B, we show

$$\frac{\partial f^*}{\partial \gamma} = \frac{4n^3\gamma^3\theta\Omega^2}{a_P^2} + \frac{2n^3\gamma^4\theta\Omega}{a_P^2}\frac{\partial\Omega}{\partial\gamma} = \frac{n^3\gamma^3\theta\Omega}{a_P^2}\left(4\Omega + 2\gamma\frac{\partial\Omega}{\partial\gamma}\right). \text{(C1)}$$

From equation (C1), we know that the sign of $\partial f^*/\partial\gamma$ is always the same as the sign of $h \equiv 4\Omega + 2\gamma(\partial\Omega/\partial\gamma)$.

The function h can be written as

$$h = 2n^3(4-5\gamma)\gamma^2x_D^2$$
$$+4n\gamma[3+n\{-3-4\gamma+n(3-4\gamma+5\gamma^2)\}]x_Dx_P$$
$$+2[2+n\{-2-6\gamma+n(2+8\gamma^2-n\gamma(3-4\gamma+5\gamma^2))\}]x_P^2. \text{(C2)}$$

The first term on the right-hand side of equation (C2) becomes strictly positive when $\gamma<4/5$. However, the signs of the second and third terms are indeterminate. Thus, in general, we cannot find the sign of h. As in Appendix B, we check the condition under which $h>0$ is realized.

From equation (C2), regardless of the magnitudes of x_D and x_P, $h>0$ is always satisfied if the following two equations are satisfied when $\gamma<4/5$.

$$h_2 \equiv 3+n\{-3-4\gamma+n(3-4\gamma+5\gamma^2)\} \geq 0, \text{(C3)}$$

$$h_3 \equiv 2+n\{-2-6\gamma+n(2+8\gamma^2-n\gamma(3-4\gamma+5\gamma^2))\} \geq 0. \text{(C4)}$$

When $\gamma<4/5$, satisfying equations (C3) and (C4) is the sufficient condition to realize $h>0$.

First, consider equation (C3). We find that equation (C3) is a convex quadratic function of γ and one minimum value exists because $3-4\gamma+5\gamma^2$ is always strictly positive. Thus, it is sufficient to check the condition for satisfying equation (C3) when the minimum value is realized. First order condition of equation (C3) with respect to γ is given by

$$\frac{\partial h_2}{\partial\gamma} = -2n\{2+n(2-5\gamma)\}. \text{(C5)}$$

From equation (C5), we know $\partial h_2/\partial\gamma<0$ is always satisfied when $n\leq 2/3$. Thus, we find that corner solution $\gamma=1$ is realized when $n\leq 2/3$.[10] Substituting $\gamma=1$ in equation (C3), we get

$$h_2 = 3-7n+4n^2. \text{(C6)}$$

Equation (C6) is a convex quadratic function of n. The minimum value of equation (C6) is realized when $n=7/8$, but it is not consistent with $n\leq 2/3$. Thus, we know that equation (C6) is a monotone decreasing function of n in the range $(0,2/3)$ and equation (C6) realizes its minimum value when $n=2/3$. Substituting $n=2/3$ in equation (C6), we get $h_2=1/9$. Thus, we find $h_2\geq 0$ is always satisfied when $n\leq 2/3$.

In contrast, from equation (C5), we can derive the following interior solution for satisfying $\partial h_2/\partial\gamma=0$

when $n \geq 2/3$.

$$\gamma = \frac{2(1+n)}{5n}. \text{(C7)}$$

Substituting equation (C7) in equation (C3), we get

$$h_2 = \frac{1}{5}\{11 + n(-23 + 11n)\}. \text{(C8)}$$

Solving $h_2=0$ in equation (C8), the following solution is derived.[11]

$$n = \frac{1}{22}(23 - 3\sqrt{5}).$$

Further, equation (C8) is a convex quadratic function of n and we find that $\partial h_2/\partial n=0 \Rightarrow n=23/22$ is the minimum value of equation (C8). Thus, we find that equation (C8) is a monotone decreasing function of n in the range [2/3,1). From the above investigation, we find that $h_2 \geq 0$ is always realized when $n \leq (23-3\sqrt{5})/22$. Finally, we find that $n \leq (23-3\sqrt{5})/22$ is the condition in which $h_2 \geq 0$ is realized.

Next, consider h_3. Differentiating equation (C4) with respect to γ, we get

$$\frac{\partial h_3}{\partial \gamma} = -n[6 + n\{3n - 8\gamma(2+n) + 15n\gamma^2\}]. \text{ (C9)}$$

Second order condition is given by

$$\frac{\partial^2 h_3}{\partial \gamma^2} = 2n^2(8 + 4n - 15n\gamma). \text{ (C10)}$$

From equation (C9), we can derive the following value for γ that satisfies $\partial h_3/\partial \gamma=0$.

$$\gamma = \frac{4(2+n) - \sqrt{-26 + 64n - 29n^2}}{15n}. \text{ (C11)}$$

Equation (C11) is a candidate for realizing the minimum value of equation (C4) because $\partial^2 h_3/\partial \gamma^2 = 2n^2\sqrt{-26+64n-29n^2} > 0$.

In relation to equation (C11), we firstly prove the following lemma.

> **Lemma C1**: Equation (C11) is always strictly positive if equation (C11) is real.

Proof:

It is clear that the denominator of equation (C11) is always strictly positive. Thus, we only check the sign of the numerator of equation (C11). $-26+64n-29n^2$ is a concave quadratic function of n and it has one maximum value.[12] The maximum value of $-26+64n-29n^2$ is realized when $n=32/29$, but it is beyond the range (0,1). Thus, the corner solution $n=1$ is realized and it provides the maximum value of $-26+64n-29n^2$. Then, substituting $n=1$ in $\sqrt{-26+64n-29n^2}$, we find that $\sqrt{-26+64n-29n^2}=3$ and it is always smaller than $4(2+n)$. Thus, the numerator of equation (C11) is always strictly positive if equation (C11) is real. *Q. E. D*

However, we notice that there are two possible situations in which equation (C11) does not realize the minimum value of equation (C4). The first possibility is that the value of equation (C11) becomes imaginary when $-26+64n-29n^2<0$. By solving $-26+64n-29n^2=0$, we get

$$n = \frac{1}{29}(32 \pm 3\sqrt{30}). \text{ (C12)}$$

From equation (C12), we know that equation (C11) becomes real only if $(32-3\sqrt{30})/29 < n < (32+3\sqrt{30})/29$ because $-26+64n-29n^2$ is a concave quadratic function of n. Thus, equation (C11) is imaginary when $n<(32-3\sqrt{30})/29$ because $(32-3\sqrt{30})/29<1<(32+3\sqrt{30})/29$. In the case of $n<(32-3\sqrt{30})/29$, equation (C9) is always negative because equation (C11) is imaginary and equation (C9) is a concave quadratic function of γ. Thus, $\gamma=1$ becomes the minimum value of equation (C4) in the case of $n<(32-3\sqrt{30})/29$.

The second possibility is that the value of equation (C11) might be more than one if $n \geq (32-3\sqrt{30})/29$, which is related to the following lemma.

> **Lemma C2**: In case $n \in [((32-3\sqrt{30})/29, 3/5)$, $\gamma=1$ realizes the minimum value of equation (C4). In contrast, when $n \geq 3/5$, equation (C11) realizes the minimum value of equation (C4).

Proof:

By using equation (C11), the following function is introduced.

$$s \equiv 15n - \left\{4(2+n) - \sqrt{-26 + 64n - 29n^2}\right\}. \text{ (C13)}$$

The condition in which $\gamma<1$ realizes the minimum value of equation (C4) is equivalent to $s>0$ and vice versa. First order condition of equation (C13), with respect to n is

$$\frac{\partial s}{\partial n} = 11 + \frac{32 - 29n}{\sqrt{-26 + 64n - 29n^2}} = 0. \text{ (C14)}$$

By solving quation (C14), we get

$$n = \frac{160 + 33\sqrt{5}}{145}. \ \text{(C15)}$$

Second order condition can be computed as

$$\frac{\partial^2 s}{\partial n^2} = -\frac{270}{(-26 + 64n - 29n^2)^{\frac{3}{2}}}. \ \text{(C16)}$$

Substituting equation (C15) to equation (C16), we get

$$\frac{\partial^2 s}{\partial n^2} = -50\sqrt{5} < 0. \ \text{(C17)}$$

Thus, we find that equation (C13) is maximized when $n=(160+33\sqrt{5})/145$. Then, we find that s is a monotone increasing function of n in the range (0,1) because $n=(160+33\sqrt{5})/145$ is more than one.

By using equation (C13), the solution of $s=0$ can be shown to be $n=3/5$. Thus, $s<0 \Rightarrow \gamma>1$ is satisfied when $n \in [(32-3\sqrt{30})/29, 3/5)$. In this case, the corner solution $\gamma=1$ realizes the minimum value of equation (C4). In contrast, $s \geq 0 \Rightarrow \gamma \leq 1$ is satisfied when $n \geq 3/5$. In this case, equation (C11) realizes the minimum value of equation (C4).

Q. E. D.

By summarizing the above discussion, we conclude that $\gamma=1$ realizes the minimum value of equation (C4) when $n<3/5$, while equation (C11) realizes the minimum value of equation (C4) when $n \geq 3/5$.

First, consider the case of $n<3/5$. Substituting $\gamma=1$ to equation (C4), we find

$$h_3 = 2(1-n)^2(1-2n). \ \text{(C18)}$$

From equation (C18), we know that $h_3 \geq 0$ is realized when $n<1/2$, while $h_3<0$ is realized when $n \in (1/2, 3/5)$.

Next, consider the case when $n \geq 3/5$. Substituting equation (C11) in equation (C4), h_3 can be depicted by Mathematica, as shown in Figure C1.[13] From Figure C1,

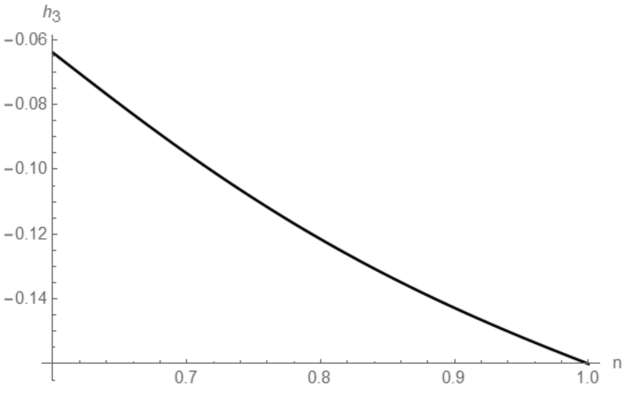

Figure C1: The function form of h_3 when $n \geq 3/5$

we know $h_3<0$ is always satisfied when $n \geq 3/5$. Thus, when $n \in (0,1)$, we find $n<1/2$ is the condition whereby $h_3>0$ is always realized.

In the end, we find that equations (C3) and (C4) are always strictly positive when $n<1/2$ because $1/2<(32-3\sqrt{30})/29$.

Finally, we know that both $n<1/2$ and $\gamma<4/5$ are sufficient conditions for realizing $\partial f^*/\partial\gamma>0$.

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[1] Winter (2013) is an excellent survey of the moral hazard in insurance economics.

[2] The website of JTSA is http://www.jtsa.or.jp/ (only in Japanese; last accessed on November 3, 2019).

[3] By definition, e_D^N never appears in this model.

[4] Second order conditions are always satisfied because $\partial^2 c^A/\partial {e_D^A}^2 = \gamma a_D > 0$, $\partial^2 c^A/\partial {e_P^A}^2 = (1-\gamma)a_P/\sqrt{\theta n f} > 0$, and $\partial^2 c^N/\partial {e_P^N}^2 = a_P/\sqrt{\theta n f} > 0$.

[5] Second order condition is always satisfied because $\partial^2 c/\partial f^2 = n^4\gamma^2\theta^2\Omega/2\theta n f^*\sqrt{\theta n f^*}a_P > 0$.

[6] Proof of $C>0$ is provided in Appendix A.

[7] Although there is another solution given by $\gamma=(139+12\sqrt{15}/131$, it is not appropriate because $\gamma>1$.

[8] Actually, the right-hand side of equation (B16) becomes

non-positive when γ is relatively large.

[9] g_3 becomes a very complex function and it is impossible to derive the characteristics of g_3 by algebraic methods.

[10] In this discussion, the range $\gamma \in [0,1]$ is used, regardless of the sign on the first term in the right-hand side of equation (C2).

[11] Although there is another solution given by $n=(23+3\sqrt{5})/22$, it is not appropriate because $(23+3\sqrt{5})/22>1$.

[12] The possibility to realize an imaginary solution will be discussed in a later section.

[13] h_3 becomes a very complex function and it is impossible to derive the characteristics of h_3 by algebraic methods.